IN THE SPECIFICATION:

Please amend paragraph number **[0024]** as follows:

[0024] ~~Fig.~~ FIG. 1 is a pictorial diagram of a computer system which comprises a presently preferred embodiment of the apparatus according to the invention, and upon which the presently preferred version of the inventive method may be carried out.

Please amend paragraph number **[0025]** as follows:

[0025] ~~Fig.~~ FIG. 2 is an illustrative example of a particle pack construction according to the preferred implementation, wherein spherical particles are placed along the central string.

Please amend paragraph number **[0026]** as follows:

[0026] ~~Fig.~~ FIG. 3 shows another illustrative example of a particle pack construction according to the preferred implementation, wherein spherical particles are placed along the central string, but wherein the particles extend more several particle diameters out from the central string.

Please amend paragraph number **[0027]** as follows:

[0027] ~~Fig.~~ FIG. 4 shows a cylindrical space with cylindrical subspaces therein according to the preferred embodiment and the preferred method of the invention.

Please amend paragraph number **[0028]** as follows:

[0028] ~~Fig.~~ FIG. 5 shows an illustrative particle pack constructed using the preferred implementation.

Please amend paragraph number **[0031]** as follows:

[0031] A ~~particle~~ "particle," as the term is used ~~herein~~ herein, is used according to its common but broad meaning in the field. It includes any type of particle that might be found in a

particle rigid body of arbitrary shape containing materials. In the preferred implementation, particles comprise rigid or nearly rigid objects of arbitrary size and shape.

Please amend paragraph number **[0034]** as follows:

[0034] ~~The~~ Illustrated in FIG. 1, the apparatus according to the invention preferably comprises a computer system 100, such as a commercially available personal computer, small business computer, engineering work station, mainframe, microprocessor, or virtually any other machine with similar processing power, appropriately configured and programmed with computer software or "code" as outlined and described herein for performing the tasks identified herein. The computer system 100 includes a processor 102, storage in the form of RAM 104, a hard drive 106, a compact disk (CD) drive 108, and a drive for diskettes 110, all contained within a cabinet 112. System 100 also includes a monitor 116, a keyboard 118, and a pointing device 120 such as a mouse, track ball or the like. System 100 also may be part of a network, in which case it may include a network connection 122 and network bus 124. The method according to the present invention, in its presently preferred implementations, may be carried out on computer system 100, using the same hardware and software, as described herein with respect to the preferred embodiment of the apparatus according to the invention.

Please amend paragraph number **[0035]** as follows:

[0035] ~~"Machine"~~ "Machine," as the term is used ~~herein~~ herein, refers broadly to a computer, processor, microprocessor, a network of such devices, or other apparatus capable of performing processing as described herein. Such machines typically and preferably will comprise a computer, such as computer system 100.

Please amend paragraph number **[0037]** as follows:

[0037] The architecture of the preferred implementation is outlined below. In its presently preferred form, it is written in double precision C++ or Fortran 90. This is not,

however, limiting. The code may, for example, be written in FORTRAN, or other suitable programming languages. An example of such a computer system is shown in ~~Fig.~~ FIG. 1. The code in its preferred version builds a single particle pack and outputs user-specified pack microgeometry and statistics. It is modular and can be disassembled into its component subroutines for use elsewhere.

Please amend paragraph number **[0090]** as follows:

[0090] The space and subspaces are constructions that define the regions or volumes into which the respective particles are to be placed. The space may comprise any contained geometric region, but preferably comprises a cylindrical construction. The subspaces also may comprise any contained geometric region, but preferably have a shape and configuration that corresponds to the overall space. In the preferred versions of the apparatus and method, the space comprises a cylindrical volume, and the subspaces comprise a plurality of concentric cylinders, disposed about a longitudinal or z axis, wherein sequential ones of the cylinders have a radius that is larger than the prior cylinders. An illustrative example of such a concentric cylinder configuration is shown in ~~Fig.~~ FIG. 2. The number of cylinders in the preferred implementation is equal to the number of particle categories, e.g., submodes, that will be present in the particle pack. The radius of each cylinder is selected to correspond to the radii of the particles. The radius of the first cylinder corresponds to or is a function of the particle radius of the smallest mode, the radius of the second cylinder is equal to the particle radius of the second smallest mode, and so on. The cylinder radius for a particular particle category preferably corresponds to or is a function of the radius or a representative radius of the corresponding particle for that mode or submode. The cylinder radius may be any positive number multiple of the particle radius. Preferably, but optionally, the cylinder radius is about 3 to 10 times the particle radius, and more preferably, about 5 to 10 times the particle radius.

Please amend paragraph number **[00177]** as follows:

[00177] In the presently preferred implementation, a catch net is incorporated into the space, below which the south pole of the falling particles cannot descend. When a particle's south pole hits the catch net belonging to its cylinder, it is placed in the pack at that point, even if it is not touching any other particle. Preferably there is a separate catch net for each particle submode, and thus for each subspace or cylinder. Each catch net preferably covers the entire ~~cross section~~ cross-section of that submode's cylinder. If the current sphere's south pole hits that catch net, it is placed there. It does not see or respond to any other submode's catch net. The altitude of the catch net preferably is determined dynamically, that is, it can depend on which placed particles are being contacted.

Please amend paragraph number **[00181]** as follows:

[00181] In the presently preferred implementation, the catch net extends across the ~~cross sectional~~ cross-sectional area of the space, across the ~~cross sectional~~ cross-sectional areas of each of the subspaces.

Please amend paragraph number **[00182]** as follows:

[00182] In this preferred implementation, the catch net comprises N subnets, one of the subnets corresponding to each of the subspaces. Each of the concentric cylinders 1, 2, 3, . . . N has a circular ~~cross section~~ cross-section perpendicular to the central string. In the preferred implementation, the catch net comprises a plurality N of subnets, or individual catch nets, one for each of the cylinders. Each of the subnets preferably extends over the ~~cross sectional~~ cross-sectional area of the corresponding subspace or cylinder. The position or level of the catch net or subnet for each of these subspaces can and typically will differ from one another, although in some instances they may be at the same or nearly the same level.

Please amend paragraph number **[00213]** as follows:

[00213] In the presently preferred implementation, the water level altitude for the *i*-th cylinder ($i > 1$) is defined as the average exposed north pole altitude of all particles with radius less than *ai*. The user may raise or lower this level by adding a user-defined offset, but the default for this offset in this implementation is zero. This water level exists between the *i*-th cylinder wall of radius *Wi* and the next smaller cylinder wall of radius W_{i-1}. ~~Thus~~ Thus, the spaces between different cylinder walls have different water levels. The cylinder corresponding to the smallest particle's submode has no water level, only a catch net.

Please amend paragraph number **[00214]** as follows:

[00214] If a larger sphere is descending onto the pack near the center cylinder, it may hit the center cylinder's pack of small particles and tend to roil off its edge and fall to the floor of the pack or fall until it hits another large sphere. ~~Thus~~ Thus, the pack would segregate with larger particles outside smaller particles. With the water level in place, however, if the larger sphere rolls off the pack in the central cylinder, it simply "floats" on the water level which is at the same level as the "surface" of the central cylinder pack. Thus, any time a descending sphere hits a water level, it is placed where it is, just as was done with the catch net. Ordinarily, water levels will be higher than catch nets, although this is not always true.

Please amend paragraph number **[00217]** as follows:

[00217] A pack sphere, e.g., the *j*-th sphere, is a candidate for being the first object encountered by the descending current sphere if their two radii overlap, *i.e.*, if

$$(X_c - X_j)^2 + (Y_c - Y_j)^2 < (a_c + a_j)^2. \qquad (35)$$

If there were no other obstacles in the way, the current sphere would strike Sphere *j* at an altitude of

$$Z_c(j) = Z_j + [(a_c + a_j)^2 - (X_c - X_j)^2 - (Y_c - Y_j)^2]^{1/2}. \tag{36}$$

Please amend paragraph number **[00251]** as follows:

[00251] The solutions are

$$s = \frac{-B \pm \sqrt{B^2 - AC}}{A} \tag{68}$$

If B^2-AC is negative, there is no solution. If it is zero, there is only one real solution which means the roll corridor just grazes the l-th cylinder, in which case it is considered a miss. If it is positive, there are two real solutions. The preferred implementation solves for both of them, and checks to determine whether either one lies in the open interval [0, 1]. If one or both lie in this interval, the l-th mode spheres are candidates for touching the descending current sphere.

Please amend paragraph number **[00267]** as follows:

[00267] The preferred implementation also can find which water level the current sphere might hit. Water level altitudes between adjacent cylinder walls are generally different as the pack is constructed. The only condition under which the current sphere might contact the k-th water level $Z_w(k)$ is if and only if the distance P_c of the current sphere from the global Z axis lies between cylinder wall radii W_{k-1} and W_k at the point of contact. P_c is given by

$$P_c = (X_c^2 + Y_c^2)^{1/2} \tag{83}$$

where X_c and Y_c can be obtained from Eq. 80 as

$$X_c = X_i + (a_c + \alpha_i)(-\cos\theta_{ji} \cos\varphi_{ji} \sin\theta_c' \cos\varphi_c' + \sin\varphi_{ji} \sin\theta_c' \sin\varphi_c'$$

$$+ \sin\theta_{ji} \cos\varphi_{ji} \cos\theta_c') \tag{84a}$$

and

$$Y_c = Y_i + (\alpha_c + \alpha_i)(-\cos\theta_{ji} \sin\varphi_{ji} \sin\theta_c' \cos\varphi_c' - \cos\varphi_{ji} \sin\theta_c' \sin\varphi_c'$$
$$+ \sin\theta_{ji} \sin\varphi_{ji} \cos\theta_c')\underline{.} \tag{84b}$$

Please amend paragraph number **[00287]** as follows:

[00287] The square root has a positive sign because the difference between φ_c and φ_u can be no greater in magnitude than $\pi/2$. Again, the arccosine has two solutions: the principal value Cos^{-1} and its negative $-\mathrm{Cos}^{-1}$. The current sphere would lose contact with the upper sphere if it is at azimuthal position $\varphi_u \pm \mathrm{Cos}^{-1}$. Its true position depends on where it was initially. If its initial position <u>is</u> $\varphi_{ci} > \varphi_u$, then the principal value Cos^{-1} is used. If its initial position <u>is</u> $\varphi_{ci} < \varphi_u$, then the negative of the principal value is used $-\mathrm{Cos}^{-1}$. In the unlikely case that the initial position equals φ_u, then the current sphere is balanced on the upper and lower sphere and a direction is randomly chosen by the random number generator. The preferred implementation using these relationships is to determine where contact is lost.

Please amend paragraph number **[00298]** as follows:

[00298] Squaring both sides gives

$$(A_2 - B_2)^2 \sin^4\varphi_c' - 2B_1(A_2 - B_2) \sin^3\varphi_c' + (A_1^2 - 2A_0A_2 - 2A_2^2 + B_1^2 + 2A_0B_2 +$$
$$2A_2B_2) \sin^2\varphi_c' + 2(A_0 + A_2)B_1 \sin\varphi_c' + (A_0 - A_1 + A_2)(A_0 + A_1 + A_2) = 0\underline{.} \tag{110}$$

Please amend paragraph number **[00314]** as follows:

[00314] This equation is of the form

$$A_0 + A_1 \cos\varphi_c' + A_2 \cos^2\varphi_c' + B_1 \sin\varphi_c' + B_2 \sin^2\varphi_c' = 0 \tag{132}$$

where in this case, the coefficients are given by

$$A_0 = X_i^2 + Y_i^2 + 2(a_c + a_i)\cos\theta_c'(X_i \sin\theta_{ji}\cos\varphi_{ji} + Y_i \sin\theta_{ji}\sin\varphi_{ji}) + (a_c + a_i)^2 \cos^2\theta_c' \sin^2\theta_{ji} \tag{133a}$$

$$A_1 = -2(a_c + a_i)\cos\theta_{ji}\sin\theta_c' [X_i \cos\varphi_{ji} + Y_i \sin\varphi_{ji} + (a_c + a_i)\cos\theta_c' \sin\theta_{ji}] \tag{133b}$$

$$A_2 = (a_c + a_i)^2 \cos^2\theta_{ji} \sin^2\theta_c' \tag{133c}$$

$$B_1 = 2(a_c + a_i)\sin\theta_c' \{\sin\theta_{ji} [X_i + (a_c + a_i)\cos\theta_c' \sin\theta_{ji}\cos\varphi_{ji}] - \cos\theta_c'[Y_i + (a_c + a_i)\cos\theta_c' \sin\theta_{ji}\sin\varphi_{ji}]\} \tag{133d}$$

$$B_2 = (a_c + a_i)^2 \sin^2\theta_c'\underline{.} \tag{133e}$$

Please amend paragraph number **[00326]** as follows:

[00326] In this expression, W_c is the radius of the cylinder wall, $P_i = \sqrt{X_i^2 + Y_i^2}$ is the radial position of Sphere i from the global Z axis, and a_c and a_i are the radii of the current sphere and Sphere i, respectively. The plus sign is used in Eq. 138 if the current sphere is rolling toward increasing Φ, the minus sign is used if the current sphere is rolling toward decreasing Φ. If the absolute value of the quantity in the parentheses of Eq. 139 is greater than unity, then there is no solution and the current sphere simply rolls around the perimeter of the cylinder until it reaches the lowest point it can. This lowest point is opposite the azimuthal position of Sphere i. ~~Hence~~ Hence, if

$$\left|\frac{W_c^2 + P_i^2 - (a_c + a_i)^2}{2W_c P_i}\right| > 1 \tag{140}$$

then

$$\Delta\Phi_c = \pm\pi \ . \tag{141}$$

Please amend paragraph number **[00330]** as follows:

[00330] The current sphere will impact a placed pack sphere, say the *j*-th sphere, if there is a solution to

$$\left|\vec{R}_c(\Phi_c) - \vec{R}_j\right| = a_c + a_j \tag{142}$$

anywhere in the range (Φ_{c0}, Φ_{cf}). The Cartesian components of the vector $\vec{R}_c(\Phi_c) = X_c, Y_c, Z_c$ are given by

$$X_c(\Phi_c) = W_c \cos\Phi_c \tag{143}$$

$$Y_c(\Phi_c) = W_c \sin\Phi_c \underline{\ .\ } \tag{144}$$

Please amend paragraph number **[00345]** as follows:

[00345] When the pack is still small enough that an insufficient number of particles have been placed to satisfactorily define a surface, the catch net may be ratcheted up a little after the placement of each particle so that the particles on the bottom of the cylinders do not all have the same altitude. If the switch ROUGHNET is true, then the current sphere's catch net moves upward by a small random amount

$$ra_{min}/N_{cs} \tag{157}$$

where r is a uniformly generated random number in [0, 1] and N_{cs} is the total number of sphere ~~cross sections~~ cross-sections required to cover all the cylinders:

$$N_{cs} = \sum_{k=1}^{n} \frac{W_k^2 - W_{k-1}^2}{a_k^2} \tag{158}$$

where n is the number of modes, W_k is the cylinder wall of the k-th mode, and $W_0 \equiv 0$. If ROUGHNET is false, then the catch net stays where it is until enough particles have been defined to make a surface and until the surface is more than $Z_{surface} - \gamma\alpha_k$ above the starting place.

Please amend paragraph number **[00362]** as follows:

[00362] The sphere is completely outside the cylinder

$$V_{in} = 0 \tag{162}$$

only if one of the following conditions are met:

If the sphere's north pole is lower than the cylinder bottom . . .

$$z_s + \alpha_s \leq z_b \tag{163a}$$

~~Or if~~ or if the sphere's south pole is above the cylinder top . . .

$$z_s - \alpha_s \geq z_t \tag{163b}$$

~~Or if~~ or if the sphere is more than its radius away from the cylinder side . . .

$$\rho_s - \alpha_s \geq \alpha_c \tag{163c}$$

~~Or if~~ or if the sphere center is above the cylinder top and is more than its radius away from the upper cylinder edge. . .

$$(\rho_s - a_c)^2 + (z_s - z_t)^2 \geq a_s^2 \text{ and } z_s > z_t \tag{163d}$$

~~Or if~~ or if the sphere center is below the cylinder bottom and is more than its radius away from the lower cylinder edge . . .

$$(\rho_s - a_c)^2 + (z_b - z_s)^2 \geq a_s^2 \text{ and } z_s < z_b . \tag{163e}$$

Please amend paragraph number **[00363]** as follows:

[00363] The sphere is completely inside the cylinder

$$V_{in} = (4/3)\pi a_s^3 \tag{164}$$

only if all three of the following conditions are simultaneously met:

If the sphere's north pole is no higher than the cylinder top . . .

$$z_s + a_s \leq z_t \tag{165a}$$

~~And~~ and if the sphere's south pole is no lower than the cylinder bottom . . .

$$z_s - a_s \geq z_b \tag{165b}$$

~~And~~ and if the sphere's equator nowhere exceeds the cylinder radius . . .

$$\rho_s + a_s \leq a_c . \tag{165c}$$

Please amend paragraph number **[00364]** as follows:

[00364] The cylinder is completely inside the sphere

$$V_{in} = \pi a_c^2(z_t - z_b) \tag{166}$$

if both the following two conditions are met:

~~If the~~ if the upper edge of the cylinder is inside the sphere . . .

$$(\rho_s + a_c)^2 + (z_t - z_s)^2 \leq a_s^2 \tag{167a}$$

~~And~~ and if the lower edge of the cylinder is inside the sphere . . .

$$(\rho_s + a_c)^2 + (z_s - z_b)^2 \leq a_s^2 . \tag{167b}$$

Please amend paragraph number **[00365]** as follows:

[00365] There are several subsets of Case 4. To efficiently organize them, we define two circles: the sphere circle and the cylinder circle. The sphere circle is the perimeter of the sphere's ~~cross section~~ cross-section at a given z altitude. Its radius is

$$r_s = [a_s^2 - (z - z_s)^2]^{1/2}, \quad z_s - a_s \leq z \leq z_s + a_s \tag{168}$$

and its radial position ρ_s is given by Eq. 161. ~~Likewise~~ Likewise, the cylinder circle is the perimeter of the cylinder's ~~cross section~~ cross-section which has constant radius a_c at any z altitude between z_b and z_t and is centered on the z axis.

Please amend paragraph number **[00388]** as follows:

[00388] Lastly,

$$\begin{aligned} E = {} & A/3\,[(A^2 + B^2)\,\mathrm{E}(\lambda_1,q) - 2B^2\,\mathrm{F}(\lambda_1,q)] - \zeta_1/3\,[(A^2 - \zeta_1^2)(\zeta_1^2 - B^2)]^{1/2} - \\ & A/3\,[(A^2 + B^2)\,\mathrm{E}(\lambda_2,q) - 2B^2\,\mathrm{F}(\lambda_2,q)] - \zeta_2/3\,[(A^2 - \zeta_2^2)(\zeta_2^2 - B^2)]^{1/2}, \quad \zeta_2 \leq A\underline{.} \end{aligned} \tag{202}$$

Again, the entire second line of this expression is zero if $\zeta_2 = A$.

Please amend paragraph number **[00394]** as follows:

[00394] The sphere is completely outside the cylinder

$$V_{in} = 0 \tag{212}$$

only if one of the following conditions are met:

If the sphere's north pole is lower than the cylinder bottom . . .

$$z_s + \alpha_s \leq z_b \tag{213a}$$

~~Or if~~ <u>or if</u> the sphere's south pole is above the cylinder top . . .

$$z_s - \alpha_s \geq z_t \tag{213b}$$

~~Or if~~ <u>or if</u> the sphere is more than its radius away from the cylinder side . . .

$$\rho_s - \alpha_s \geq \alpha_c \tag{213c}$$

~~Or if~~ <u>or if</u> the sphere center is above the cylinder top and is more than its radius away from the upper cylinder edge . . .

$$(\rho_s - \alpha_c)^2 + (z_s - z_t)^2 \geq \alpha_s^{\;2} \quad \text{and} \quad z_s > z_t \quad \text{and} \quad \rho_s > \alpha_c \tag{213d}$$

~~Or if~~ <u>or if</u> the sphere center is below the cylinder bottom and is more than its radius away from the lower cylinder edge . . .

$$(\rho_s - \alpha_c)^2 + (z_b - z_s)^2 \geq \alpha_s^{\;2} \quad \text{and} \quad z_s < z_b \quad \text{and} \quad \rho_s > \alpha_c\,. \tag{213e}$$

Please amend paragraph number **[00395]** as follows:

[00395] The sphere is completely inside the cylinder

$$V_{in} = (4/3)\pi\alpha_s^3 \quad (214)$$

only if all three of the following conditions are simultaneously met:

~~If the~~ if the sphere's north pole is no higher than the cylinder top . . .

$$z_s + \alpha_s \leq z_t \quad (215a)$$

~~And if~~ and if the sphere's south pole is no lower than the cylinder bottom . . .

$$z_s - \alpha_s \geq z_b \quad (215b)$$

~~And if~~ and if the sphere's equator nowhere exceeds the cylinder radius . . .

$$\rho_s + \alpha_s \leq \alpha_c \ . \quad (215c)$$

Please amend paragraph number **[00396]** as follows:

[00396] The cylinder is completely inside the sphere

$$V_{in} = \pi\alpha_c^2(z_t - z_b) \quad (216)$$

if both the following two conditions are met:

~~If the~~ if the upper edge of the cylinder is inside the sphere . . .

$$(\rho_s + \alpha_c)^2 + (z_t - z_s)^2 \leq \alpha_s^2 \tag{217a}$$

~~And if~~ and if the lower edge of the cylinder is inside the sphere . . .

$$(\rho_s + \alpha_c)^2 + (z_s - z_b)^2 \leq \alpha_s^2 \ . \tag{217b}$$

Please amend paragraph number **[00397]** as follows:

[00397] Again, there are several subsets of Case 4. To efficiently organize them, we define two circles: the sphere circle and the cylinder circle. The sphere circle is the perimeter of the sphere's ~~cross section~~ cross-section at a given z altitude. Its radius is

$$r_s = [\alpha_s^2 - (z - z_s)^2]^{1/2}, \quad z_s - \alpha_s \leq z \leq z_s + \alpha_s \tag{218}$$

and its radial position ρ_s is given by Eq. 211. ~~Likewise~~ Likewise, the cylinder circle is the perimeter of the cylinder's ~~cross section~~ cross-section which has constant radius α_c at any z altitude between z_b and z_t and is centered on the z axis.

Please amend paragraph number **[00402]** as follows:

[00402] Armed with these four solutions, we will now consider each of the four subcases of Case 4, but we will consider separately whether the sphere circle lies in the southern hemisphere of the sphere or whether it is the equatorial plane or northern hemisphere. We consider the four subcases separately for the two hemispheres making eight situations in all. We consider the hemispheres separately because if the sphere circle is in the southern hemisphere, it is increasing in size as z increases. ~~Hence~~ Hence, the preferred implementation can determine the altitude at which subcases change. If the sphere circle is its equatorial plane or is its northern

hemisphere, it is decreasing in size as z decreases, and the altitude at which subcases will change again can be determined.